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Photon sorters and QND detectors using single photon emitters

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We discuss a new method for realizing number-resolving and non-demolition photo detectors by strong coupling of light to individual single photon emitters, which act as strong optical non-linearities. As a specific application we show how these elements can be integrated into an error-proof Bell state analyzer, whose efficiency exceeds the best possible performance with linear optics even for a modest atom-field coupling. The methods are error-proof in the sense that every detection event unambiguously projects the photon state onto a Fock or Bell state.

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Experimental realization of number-resolving, non-demolition photo (QND) detectors is a long-standing challenge in quantum optics and quantum information science. Conventional photodetectors measure only the intensity or the energy of an incoming light pulse, and are not capable of measuring photon states in a QND fashion. More advanced measurements schemes can be constructed using optical non-linearities, but these are typically very weak since photons rarely interact with each other. In this Rapid Communication, we show how to overcome this problem by strong coupling of light to individual single photon emitters. This provides a strong optical non-linearity, which enables the realization of number-resolving photon sorters and quantum non-demolition photo detectors.

A common approach to realizing strong coupling between photons and atoms relies on cavity QED, where the light field is confined to a high–Q resonator [1–3]. In order to reach the strong coupling regime in small integrated devices, great advances have been made using photonic crystals [4, 5], tapered optical fibers coupled to a single atom [6, 7], microwave transmission lines coupled to a flux qubit [8], or surface plasmons modes coupled to a single photon emitter [9–11]. In these systems the emitter couples to a continuous one-dimensional spectrum of modes and photon scattering is governed by the interference of absorbed, reemitted, and directly transmitted waves [12–15]. In a similar way, the transmission of a tighly focussed light beam can be controlled by a single emitter in free space [16, 17]. In the present paper we will explore possible applications of emitters coupled to such a one-dimensional photonic continuum for photo detection, but the ideas and formalism we use can also be applied to cavity QED as well as other methods of achieving strong optical non-linearities [18–20].

First, we consider passive devices based on simple two-level emitters. The interaction with the emitter naturally leads to a photon turnstile effect, which can be used to implement a number resolving photon sorter. Secondly, we consider a waveguide coupled to a three-level emitter controlled by a classical laser field. This setup offers significantly more opportunities at the expense of a more complex optical setup. In particular we discuss QND photo detectors. As a possible applications of these devices we will show how to construct optical Bell-state analyzers. A Bell measurement is an essential ingredient in quantum information, as it enables efficient quantum repeaters [21] as well as universal optical quantum computers [22]. Unfortunately such a measurement cannot be realized with linear optics [23], but requires a strong nonlinearity. We focus on realistic systems with losses and discuss how to make devices error-proof. Even in the case of an error, the measurement shall at most give an inconclusive but never a wrong result.

One of the conceptually simplest extensions of linear optics is a device capable of non-destructively distinguishing single and two photon pulses. Such a photon sorter can be realized with only passive optical elements and simple two level emitters using the setup sketched in Fig. 1. The procedure is most easily explained if the emitters are coupled to semi-infinite waveguides extending only in one direction, but can also be realized with infinite waveguides if more optical elements are used [11, 15]. We assume that the incoming pulse enters the interferometer in the upper arm labelled by $\hat{a}_{\rm in}$. A single photon is split at the beam splitter and brought to interact with the emitters, where it experiences a phase shift [12]

$$t_k = \frac{ck - \hbar\omega_0 + i\hbar(\gamma - \Gamma)/2}{ck - \hbar\omega_0 + i\hbar(\gamma + \Gamma)/2}\,, \qquad (1)$$

where k is the photon wavenumber. Similar to the quantum jump approach [24] we have added an imaginary part to the resonance frequency of the emitter $\omega_0 - i\gamma/2$ describing a coupling of the emitter to other modes than the waveguide with a rate γ. The coupling strength is given by the rate of spontaneous emission into the waveguide Γ. The phase shift (1) depends on the wavenumber k but is the same in both arms of the interferometer. The interferometer can therefore be balanced such that a single photon always leaves the setup in mode $\hat{a}_{\rm out}$. This is

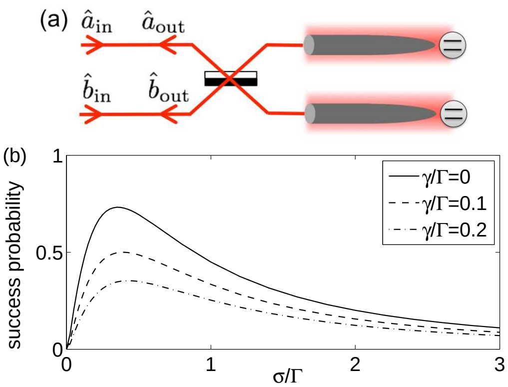

FIG. 1: (a) A photon sorter based on a 1D waveguide end-coupled to a single emitter. The incident photons (mode $\hat{a}_{\rm in}$) are split at a beam splitter and both modes then interact with a single emitter. A single photon will always be emitted in mode $\hat{a}_{\rm out}$ while two photons are likely to be emitted in mode $\hat{b}_{\rm out}$ but never split into one photon in each arm. (b) The success probability of the photon sorter, i.e. the probability that two photons are scattered to the mode $\hat{b}_{\rm out}$, as a function of the frequency width σ of a Gaussian input pulse.

different if two photons enter the setup and interact indirectly via the emitter in which case they can leave the interferometer in mode $\hat{b}_{\rm out}$ with a significant probability. To be more precise, we consider an input state of two identical photons with pulse shape $f_2(k,p) = f_1(k)f_1(p)$:

$$|\Psi_{\rm in}\rangle = \frac{1}{\sqrt{2}}\int dk\, dp\, f_2(k,p)\hat{a}_k^\dagger \hat{a}_p^\dagger|0\rangle. \qquad (2)$$

The beam splitter mixes the modes $\hat{a}$ and $\hat{b}$ such that $\hat{a}_k^\dagger\hat{a}_p^\dagger \to \hat{a}_k^\dagger\hat{a}_p^\dagger + 2\hat{a}_k^\dagger\hat{b}_p^\dagger + \hat{b}_k^\dagger\hat{b}_p^\dagger$. When the photons then interact with the same emitter, this introduces a strongly correlated 'bound state' contribution

$$f_1(k)f_1(p) \to t_k t_p f_1(k)f_1(p) + f_B(k,p), \qquad (3)$$

whose precise form can be found in Refs. [13, 14]. Finally, after interacting with the beam splitter once again, the two-photon input state (2) is transformed to

$$|\Psi_{\rm out}\rangle = \frac{1}{2\sqrt{2}}\int dkdp\, f_B(k,p)\hat{b}_k^\dagger\hat{b}_p^\dagger|0\rangle \qquad (4)$$
$$+\frac{1}{\sqrt{2}}\int dkdp\left(t_k t_p f_1(k)f_1(p) + \frac{1}{2}f_B(k,p)\right)\hat{a}_k^\dagger\hat{a}_p^\dagger|0\rangle.$$

Note that there are no mixed terms (e.g. $\hat{a}_k^\dagger\hat{b}_p^\dagger$), so that the two photons always leave the interferometer in the same arm. There is a significant probability that the two photons leave the setup in mode $\hat{b}_{\rm out}$, whereas a single photon always leaves the the interferometer in mode $\hat{a}_{\rm out}$, such that the interferometer acts as a photon sorter.

The success probability of the photon sorter, i.e. the probability that two photons are scattered to the mode $\hat{b}_{\rm out}$ is given by

$$p_s = \frac{1}{4}\int dk\, dp\, \|f_B(k,p)\|^2. \qquad (5)$$

The results are shown in Fig. 1 (b) for a Gaussian input pulse $f_1(k) \sim \exp(-(ck-\hbar\omega_0)^2/4\sigma^2)$ as a function of the frequency width σ. One observes that the efficiency of the photon sorter strongly depends on the pulse shape of the incident photons. They must be resonant to the atomic transition and the frequency width σ should therefore not be too large. On the other hand the photons should be tightly localized in real space as they only interact when they are at the same position. Thus an optimum value of the efficiency is found for intermediate values of σ.

Regardless of the success probability the photon sorter can provide insight into the nature of the incoming light pulse. If for instance a conventional photo detector detects the output in mode $\hat{b}_{\rm out}$, the intensity of that measurement directly reflects the two photon contribution in the pulse. On the other hand the success probability can be increased in array of concatenated devices by feeding the output mode $\hat{a}_{\rm out}$ of one sorter to the next one. For example, an array of five photon sorter increases the success probability to 96 %. The success rate of such more complex devices will be discussed in detail elsewhere [25].

To realize more advanced photo detection schemes we shall now consider three-level emitters as also discussed in Ref. [3, 26] in the framework of cavity QED. Generalizing these approaches, we now describe how to construct a QND photo detector using the setup shown in Fig. 2 (a). A three-level emitter is prepared in a coherent superposition of the ground state $|g\rangle$ and a metastable state $|s\rangle$, which does not couple to the waveguide,

$$|g\rangle \longrightarrow \alpha|g\rangle + \beta|s\rangle, \quad \text{and} \quad |s\rangle \longrightarrow -\beta|g\rangle + \alpha|s\rangle, \quad (6)$$

with $\beta = \sqrt{1-\alpha^2}$. A passing photon then introduces a phase shift if and only if the emitter is in state $|g\rangle$. In particular the transmission amplitude on resonance is given by $t_0 = (\gamma-\Gamma)/(\gamma+\Gamma)$. Then one applies another classical control pulse which inverts the transformation (6). The complete procedure thus realizes the mapping

$$\begin{aligned} &\text{1 photon:} && |g\rangle \to (\beta^2 + t_k\alpha^2)|g\rangle + \alpha\beta(1-t_k)|s\rangle, \\ &\text{0 photons:} && |g\rangle \to |g\rangle. \end{aligned} \qquad (7)$$

If the state of the emitter is now measured to be $|s\rangle$, e.g. by measuring the transmission of a classical light pulse afterwards, this unambiguously reveals the presence of a single photon. Unlike conventional photo detection this does not disturb the photon, i.e. the scheme realizes a QND measurement. Furthermore if no photon is present, the emitter returns deterministically to the initial state $|g\rangle$, i.e. there are no dark counts. The optimal detector efficiency $\Gamma/(\gamma+\Gamma)$ approaches unity when $\Gamma \gg \gamma$. It is reached when $\alpha = 1/\sqrt{2}$. For the Bell state measurement discussed below, it is advantageous to reduce the error of detecting the emitter in state $|g\rangle$ although a photon has passed the detector. This strategy is realized by the choice $\alpha = 1/(1-t_0)$.

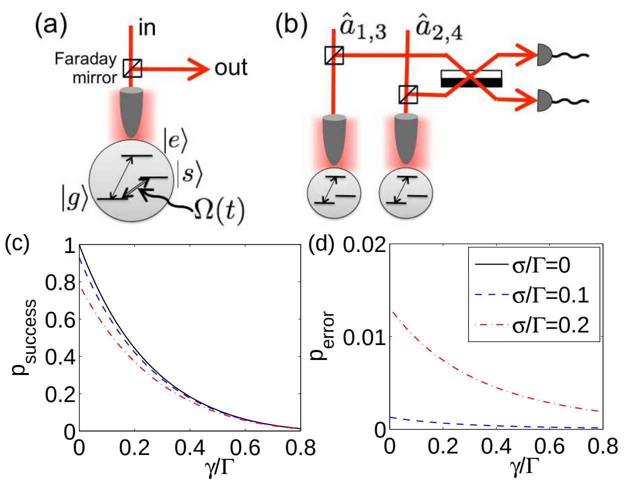

FIG. 2: (a) A QND photo detector based on three-level emitters strongly coupled to a one-dimensional waveguide. (b) Integration in a Bell state analyzer. (c) The success and error probabilities as a function of the scaled decay rate γ/Γ for a Gaussian input pulse for different values of the width σ.

The photo detection schemes discussed above may be used in a variety of contexts where the measurement of more involved properties of light is required. A particularly important application is the design of an optical Bell state analyzer, which distinguishes the four Bell states

$$|\phi^{\pm}\rangle = \frac{1}{2}\int dk\, dp\, f_1(k) f_1(p) \left(\hat{a}^{\dagger}_{1k}\hat{a}^{\dagger}_{3p} \pm \hat{a}^{\dagger}_{2k}\hat{a}^{\dagger}_{4p}\right)|0\rangle$$
$$|\psi^{\pm}\rangle = \frac{1}{2}\int dk\, dp\, f_1(k) f_1(p) \left(\hat{a}^{\dagger}_{1k}\hat{a}^{\dagger}_{4p} \pm \hat{a}^{\dagger}_{2k}\hat{a}^{\dagger}_{3p}\right)|0\rangle, \quad (8)$$

where the subscripts $1-4$ refer to four different photonic modes. In principle, a BSA can be achieved directly from the scheme for photonic quantum gates in cavity QED [3]. Such setups, however, often require rapid switching of the optical path and delay lines for photons, which is experimentally unfavorable. Here, we consider a modified version of Ref. [3], which avoids these elements and integrate it into a BSA. This setup shall be efficient and error-proof even for an imperfect coupling, i.e $\gamma \neq 0$, so that it cannot give a wrong measurement result.

For resonant input photons, the photo QND detector introduced above is sufficient to realize a simple error-proof BSA using the setup shown in Fig. 2 (b). We assume that the logical state of both control and target photon are encoded into two spatial modes. The control photon (modes $\hat{a}_{1,2}$) passes the setup well before the target photon (modes $\hat{a}_{3,4}$). For the Bell states $|\phi^{\pm}\rangle$, both photons pass the same arm of the setup subsequently. The emitter coupled to this arm is transferred from state $|g\rangle$ to $|s\rangle$ and back to state $|g\rangle$ after interacting with the control and target photon, respectively. The other emitter always remains in the internal state $|g\rangle$. On the contrary, the two photons pass through different arms of the interferometer for the Bell states $|\psi^{\pm}\rangle$, so that both emitters are transferred to the state $|s\rangle$. A measurement of the internal state of the emitters thus allows to distinguish between the subspaces spanned by $|\phi^{\pm}\rangle$ (atomic state $|gg\rangle$) on the one hand and $|\psi^{\pm}\rangle$ (atomic state $|ss\rangle$) on the other hand. Whether it is the plus or the minus sign is revealed by the coincidence pattern of detectors placed after a beamsplitter mixing the modes 1 and 2 as well as 3 and 4. Taking into account photon loss, the success probability of this BSA is given by $p_{\text{success}} = \eta^2|t_0|^2 = \eta^2(\gamma-\Gamma)^2/(\gamma+\Gamma)^2$, where η is the efficiency of the final photo detectors. With non-resonant input, the probability for a successful Bell measurement is given by

$$p_{\text{success}} = \frac{\eta^2|t_0|^2}{|1-t_0|^4}\int dk\, dp\, |f_2(k,p)(1-t_k)(1-t_p)|^2 \quad (9)$$

regardless of which of the Bell states is incident. This result (with $\eta = 1$) is plotted in Fig. 2 (c) as a function of the loss rate γ/Γ for Gaussian input pulses. One finds that a Purcell factor of $\Gamma/\gamma \approx 5.8$ is sufficient to exceed the $\eta^2 \times 50$ % limit of linear optics.

The present setup is, however, not strictly error-proof if the input photons are not completely resonant. While the measurement result $|ss\rangle$ leads to an unambiguous Bell state measurement, the result $|gg\rangle$ does not. It can be almost certainly attributed to the subspace spanned by $|\phi^{\pm}\rangle$, but there is a small probability that it has been triggered by the states $|\psi^{\pm}\rangle$. This error can, however, be suppressed to a large extend by choosing a rotation angle of $\alpha = 1/(1-t_0)$. In this case the residual probability to obtain an erroneous measurement result for the input state $|\psi^{\pm}\rangle$ is given by

$$p_{\text{error}} = \frac{\eta^2}{|1-t_0|^4}\int dkdp\, |f_2(k,p)(t_0-t_k)(t_0-t_p)|^2. \quad (10)$$

For a Gaussian wavepacket, p_{error} vanishes as $\sigma^4/(\gamma+\Gamma)^4$ for $\sigma/(\gamma+\Gamma) \to 0$. It thus remains small also for a non-monochromatic input photon as shown in Fig. 2 (d).

In order to realize a fully error-proof BSA, one needs another measurement stage, which unambiguously detects $|\phi^{\pm}\rangle$. In principle this can be realized by exchanging the modes $\hat{a}_1$ and $\hat{a}_2$ and then repeating the above scheme [25]. This would, however, require rapid switching of the optical path between control and target photon.

As we will now show, a fully passive, error-proof BSA may in fact be constructed using the photon sorters introduced above. The setup to perform these operations is summarized in Fig. 3 (a). Assume that the four optical modes containing the Bell state in Eqn. (8) are incident on a beam splitter array mixing the modes 1 and 4 as well as 2 and 3 (denoted by BS_1 in Fig. 3). The states $|\psi^{\pm}\rangle$ are mapped onto $\sim(\hat{a}_1^{\dagger 2} - \hat{a}_4^{\dagger 2} \pm \hat{a}_2^{\dagger 2} \mp \hat{a}_3^{\dagger 2})|0\rangle$, suppressing the pulse shape for simplicity. The two photons are always located in the same mode for the states $|\psi^{\pm}\rangle$, whereas they are always located in two different modes for the

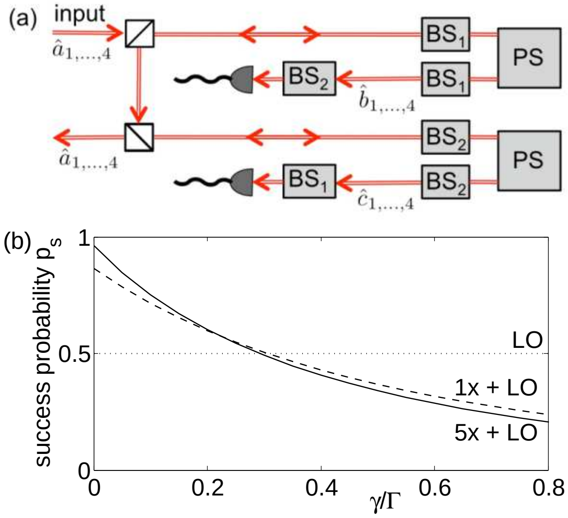

FIG. 3: (a) Setup of a BSA composed of a photon sorter (PS) and linear beam splitter arrays (BS_1 and BS_2, see text). The crossed squares represent Faraday mirrors separating incoming and reflected modes. Each of the arrows represents four modes carrying the Bell state. (b) Total success probability of an array of $n = 1$ (1x + LO) and $n = 5$ (5x+LO) error-proof BSAs, plus a linear optical BSA at the end of the array as a function of the ratio γ/Γ assuming a Gaussian pulse shape with frequency width $\sigma/\Gamma = 0.36$. The success probability of a linear optical BSA (LO) is plotted for comparison.

states $|\phi^\pm\rangle$. If each of the modes is now incident on the photon sorter introduced above, the states $|\psi^\pm\rangle$ are separated to the modes $\hat{b}_{1,...,4}$ with a significant probability. It is then possible to distinguish between $|\psi^+\rangle$ and $|\psi^-\rangle$ with linear optics and conventional photodetectors only, giving rise to an unambigious Bell state measurement. If no photon is detected we can simply go on with the two photons in the modes $\hat{a}_{1,...,4}$. In order to detect also the Bell states $|\phi^\pm\rangle$, one undoes the effect of the first beam splitter array BS_1 and then mixes the modes 1 and 3 as well as 2 and 4 instead (denoted by BS_2 in Fig. 3). Now it's the states $|\phi^\pm\rangle$, for which the two photons are located in the same mode, these are thus separated to the modes $\hat{c}_{1,...,4}$ by the photon sorter and subsequently detected. The photons are either measured in the two detector arrays projecting unambiguously onto one of the Bell states or leave the BSA in the modes $\hat{a}_{1,...,4}$. In the latter case another measurement can be attempted.

The proposed BSA works probabilistically – with a non-vanishing probability the photons are not detected but just transmitted through the complete setup. The success probability is given by the probability of two photons to be scattered to the modes $\hat{b}_{1,...,4}$ or $\hat{c}_{1,...,4}$, respectively, and thus given by the success probability of the photon sorter, which is given in Eqn. (5) and plotted in Fig. 1 (b). If the detection fails and the photons are transmitted, one can just repeat all operations. However, in actual experiments photon losses are inevitable and the coupling of the emitter to the one-dimensional waveguide is not perfect. Photon loss only leads to an inconclusive measurement result and Fig. 3 (b) shows the resulting success probability of an array of 1 (dashed line) and 5 (solid line) concatenated BSAs, as a function of the ratio γ/Γ. After passing this array, the remaining modes are detected with a linear optical BSA. As shown in the figure already a modest Purcell factor of $\Gamma/\gamma \approx 3.3$ is sufficient to exceed the 50 % limit of linear optics.

In conclusion we have shown that the coupling of single emitters to one-dimensional waveguides opens up new possibilities for number resolving, non-demolition photo detection. We have explicitly shown how to construct photon sorters and QND detectors, and that these systems can be used for efficient Bell state analysis. Most importantly the devices are error proof in the sense that imperfect coupling only leads inconclusive and not wrong results. As a consequence the devices work with modest coupling efficiencies, which are well within reach of current experiments.

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